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Savings growth and the path of utility

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Abstract. We derive an expression relating the change in instantaneous utility to the growth of net (genuine) saving in an economy with multiple stocks and externalities that maximizes welfare in the utilitarian sense. This result is then shown to hold for decentralized competitive efficient economies as well, to yield an extension of the Hartwick rule: instantaneous utility is non-declining along a development path if genuine saving is decreasing. By way of example the rule is applied as a constant genuine saving rate rule in a simple Dasgupta-Heal-Solow-Stiglitz economy. The rule yields a path with unbounded consumption and higher wealth than on the standard Hartwick constant consumption path. JEL classification: Q01, Q56, O13

Croissance de l'épargne et sentier d'utilité. L'auteur dérive une relation entre le changement d'utilité instantanée et la croissance de l'épargne nette (véritable) dans une économie à multiples stocks et externalités qui maximise le bien-être dans un sens utilitariste. On montre que ce résultat tient pour des économies compétitives efficaces décentralisées, et donc génère une extension de la règle de Hartwick: l'utilité instantanée ne décline pas au fil d'un sentier de développement si l'épargne véritable décroît. On applique la règle en tant que règle de taux constant d'épargne véritable dans une économie à la Dasgupta-Heal-Solow-Stiglitz. La règle engendre un sentier où la consommation n'est pas limitée et la richesse est plus grande que pour le sentier standard de consommation constante à la Hartwick.

1. Introduction

Recent papers by Hamilton and Clemens (1999), Dasgupta and Mäler (2000), and Hamilton and Hartwick (2005) have explored the linkage between savings,

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welfare and sustainable development.¹ Hamilton and Clemens report a result proved in Hamilton (1997) that on the optimal path for an economy with heterogeneous assets *genuine saving*² measured in utils is equal to the change in welfare (the present value of utility on the path). Dasgupta and Mäler establish the same result for a non-optimal economy with a suitable definition of the accounting prices for assets. Hamilton and Hartwick show that growth in consumption in an optimal Dasgupta-Heal-Solow-Stiglitz (DHSS) economy is related to the difference between the growth rate of genuine saving and the interest rate.

In the present paper we are interested in the relationship between the change in genuine savings and the change in instantaneous utility. Dixit, Hammond, and Hoel (1980) have established that in a competitive economy without externalities, constant genuine savings implies constant instantaneous utility. First, we extend this result to a more general economy with externalities as flows or stocks and show that, if the externalities are properly internalized, instantaneous utility increases over time if and only if genuine savings are decreasing. Second, we investigate increasing consumption in the DHSS economy. It is shown that a constant genuine saving rate rule leads to unbounded consumption, a result we relate to Dasgupta and Heal (1979) and Mitra (1983). The result also suggests that certain modes of unbounded population growth are compatible with sustainability in the model, an issue further explored in Asheim et al. (2005).

2. The model: optimality and perfect competition

In this section our approach is first to define a social optimum and subsequently to consider its implementation in a decentralized setting. We use optimal control theory, which yields the desired results in a straightforward way.

The set of stocks of the economy consists of man-made capital, human capital, non-renewable natural resources, renewable natural resources, accumulated pollutants, and so on. The corresponding n -vector of state variables is denoted by $x = (x_1, x_2, \dots, x_n)$ with initial stocks $x(0) := (x_1(0), x_2(0), \dots, x_n(0))$ given. There are m flow variables, represented by the m -vector $u = (u_1, u_2, \dots, u_m)$. It includes rates of consumption, harvesting, extraction, and emissions. The function U denotes the instantaneous utility function. It depends on the stocks, to allow for stock amenities, as well as on flows. The time path of the state variables is described by the vector function $f = (f^1, f^2, \dots, f^n)$, depending on both stocks and flows. In addition, other constraints prevail in terms of technology, balance of payments conditions, or physical relationships. These are represented by

1 We define sustainability, after Pezzey (1989), to be a property of a development path: a path is sustainable if utility is everywhere non-decreasing along it. There are other measures of sustainability, such as in Pezzey (2004), where it is required that current utility not exceed current maximum sustainable utility.

2 Hamilton and Clemens (1999) use the term 'genuine' saving to distinguish what is a net saving measure from the restricted notion of net saving employed in standard national accounting.

$g(x, u) \geq 0$, with $g = (g^1, g^2, \dots, g^q)$ a vector function, depending on stocks and instruments.

Following Dixit, Hammond, and Hoel (1980), it is assumed that all functions involved are time-independent.³ Moreover, they are ‘regular,’ in the sense of being smooth enough to allow for a standard application of optimal control theory without bothering about technicalities. For later reference we define the production possibility set:

$$Z = \{(x, \dot{x}, u) : [0, \infty) \rightarrow R^{2n+m} \mid \text{such that} \\ \dot{x}(t) = f(x(t), u(t)), g(x(t), u(t)) \geq 0, \forall t \geq 0\}.$$

We start by deriving the utilitarian optimum, defined as the solution of the following problem:

$$\max \int_0^\infty \pi(t) U(x, u) dt,$$

subject to $(x, \dot{x}, u) \in Z$. Here, π is the discount factor. The present value Hamiltonian is $H(x, u, \lambda, t) := \pi(t)U(x, u) + \lambda \cdot f(x, u)$, where λ is an n -vector of co-state variables. The present value Lagrangian is $L(x, u, \lambda, \mu, t) := \pi(t)U(x, u) + \lambda \cdot f(x, u) + \mu \cdot g(x, u)$, where μ is a q -vector of multipliers. A first necessary condition is the maximization of the present value Lagrangian with respect to the flow variables. This yields

$$\pi(t) \frac{\partial U}{\partial u_k} + \sum_{j=1}^n \lambda_j \frac{\partial f^j}{\partial u_k} + \sum_{l=1}^q \mu_l \frac{\partial g^l}{\partial u_k} = 0, \quad k = 1, 2, \dots, m. \quad (1)$$

Along an optimum the co-state variables λ evolve according to

$$\pi(t) \frac{\partial U}{\partial x_i} + \sum_{j=1}^n \lambda_j \frac{\partial f^j}{\partial x_i} + \sum_{l=1}^q \mu_l \frac{\partial g^l}{\partial x_i} = -\dot{\lambda}_i, \quad i = 1, 2, \dots, n. \quad (2)$$

Finally, the following complementary slackness conditions hold:

$$\mu_l \geq 0, \mu_l g^l(x, u) = 0, \quad l = 1, 2, \dots, q. \quad (3)$$

Define genuine saving as the present value of net investments in all stocks $G = \lambda \cdot f$. Along an optimum

$$\dot{L} = \frac{dL}{dt} = \frac{\partial L}{\partial t},$$

³ For an analysis of the complications time-dependence introduces see Vellinga and Withagen (1996).

so that $\dot{H}(x, u, t) = \dot{\pi}(t)U(x, u) + \pi \dot{U}(x, u) + \dot{G} = \dot{\pi}(t)U(x, u)$. Hence, we obtain

PROPOSITION 1. *Along a utilitarian optimum $\pi \dot{U}(x, u) = -\dot{G}$.*

Therefore, along an optimum the sign of the change in genuine savings indicates whether instantaneous utility is increasing or decreasing. This generalizes the first proposition in Hamilton and Hartwick (2005).

For a competitive economy without externalities Dixit, Hammond, and Hoel (1980) show that constant genuine savings lead to constant utility. We will derive a related result, but with two important modifications: first, we incorporate external effects, and, second, based on the result of proposition 1, we study the possibility of increasing utility over time.

Externalities can be classified in different ways. They can be negative or positive or both (at different thresholds or points in time); they can affect utility or technology or both; they can be a stock or a flow externality; and, finally, they can originate from consumption or from production or both. It is well known that in the presence of externalities the social optimum, defined above, can be realized as a competitive general equilibrium by appropriate taxation of the externalities. For the case at hand, the vector of co-state variables λ provides the value of the stocks in terms of their overall contribution to welfare, including therefore the optimal tax to be levied. We can conclude that if the government pursues social welfare maximization by means of taxation of externalities, the prices of capital will be set equal to λ . Hence, we have the following proposition:

PROPOSITION 2. *Suppose the social optimum is implemented as a general intertemporal competitive equilibrium with taxation of externalities. Then $\pi \dot{U}(x, u) = -\dot{G}$, with $G = \lambda \cdot f$ and λ the optimal price of stocks, defined above.*

Proof. In order to provide additional intuition and a simple proof, we restrict ourselves to stock externalities arising from production only and affecting utility only. Following Dixit, Hammond, and Hoel (1980), we introduce the price vectors $p = (p_1, p_2, \dots, p_m)$ and $q = (q_1, q_2, \dots, q_n)$ for the flow commodities and the stocks, respectively. In addition, we introduce a vector of taxes on stocks denoted by $\tau = (\tau_1, \tau_2, \dots, \tau_n)$. The tuple $(p, q, \tau, x, \dot{x}, u) : [0, \infty) \rightarrow R^{4n+2m}$ with $(x, \dot{x}, u) \in Z$, constitutes a *general competitive efficient equilibrium* if (omitting the time index)

$$p_k = \pi \frac{\partial U}{\partial u_k}, k = 1, 2, \dots, m, \quad \tau_i = \pi \frac{\partial U}{\partial x_i}, \quad i = 1, 2, \dots, n \quad (4)$$

$$p \cdot u + q \cdot \dot{x} + \dot{q} \cdot x + \tau \cdot x \geq p \cdot \hat{u} + q \cdot \hat{x} + \dot{q} \cdot \hat{x} + \tau \cdot \hat{x}, \text{ for all } (\hat{x}, \hat{x}, \hat{u}) \in Z \quad (5)$$

$$u \text{ maximizes } \pi U(x, u) - p \cdot u. \quad (6)$$

The equations in (4) define the optimal taxation of the externalities (including flow externalities), if applicable. Aggregate profits in (5) consist of the revenues from selling the consumer commodities, newly produced capital goods (including stocks providing externalities) at prices q , the change in value of capital goods, and the tax on capital goods because of the externality. The latter will be a tax when the capital good causes a negative externality. Finally, consumers maximize utility, which yields (6). For a detailed discussion of the conditions (5) and (6) we refer to Dixit, Hammond, and Hoel (1980, 552).

Let us define genuine savings by $G = q \cdot \dot{x}$. Starting from the definition of a competitive efficient equilibrium we now derive $\pi \dot{U} = -\dot{G}$ following the argument in Dixit, Hammond, and Hoel (1980, theorem 1). From (4) we have $\pi \dot{U} = p \cdot \dot{u} + \tau \cdot \dot{x}$. Fix time t and Δt . It follows from profit maximization that

$$\begin{aligned} p(t) \cdot u(t + \Delta t) + q(t)\dot{x}(t + \Delta t) + \dot{q}(t)x(t + \Delta t) + \tau \cdot x(t + \Delta t) \\ \leq p(t) \cdot u(t) + q(t)\dot{x}(t) + \dot{q}(t)x(t) + \tau \cdot x(t). \end{aligned}$$

Divide by Δt and let Δt go to zero, from below as well as from above. Then it follows that

$$p(t) \cdot \dot{u}(t) + \tau(t) \cdot \dot{x}(t) + \frac{d(q(t) \cdot \dot{x}(t))}{dt} = 0.$$

Obviously, it must be the case that $q = \lambda$.

This proposition is a corollary to theorem 1 in Dixit, Hammond, and Hoel (1980), who show that the Hartwick rule (Hartwick 1977) can be generalized to state that constant genuine saving (in present value prices) implies constant utility in a competitive economy. Proposition 2 shows that inherent in Dixit, Hammond, and Hoel (1980) is a more general result relating the change in instantaneous utility to the change in genuine saving. This opens the door to policy rules more general than the Hartwick rule, with the Hartwick rule as a special case.

Proposition 2 can be restated in a way that is useful for the sake of interpretation as well as for the application to be discussed in the next section. Assume that the first flow commodity u_1 occurs as a private commodity in the instantaneous utility function. Moreover, assume that $\dot{x}_1 = f^1(x, u_2, u_3, \dots, u_m) - u_1$ and u_1 does not appear in any of the other f and g functions (except possibly in a non-binding non-negativity constraint). It follows from profit maximization with respect to x_1 , say, man-made capital, that

$$r := -\frac{\dot{q}_1}{q_1} = \sum_{i=1}^n \frac{q_i \partial f^i / \partial x_1}{q_1} + \frac{\tau_1}{q_1}.$$

Hence, the rental rate r reflects total man-made capital productivity. But it includes not only marginal productivity as a production factor in the production of the consumer commodity of sector one ($\partial f^1 / \partial x_1$) and all other sectors

$(\partial f^i / \partial x_1, i = 2, 3, \dots, n)$, but also the possibly negative effects of pollution arising from the use of man-made capital. We can now write

$$\pi \dot{U} = -q_1 \tilde{G} \left(\frac{\dot{\tilde{G}}}{\tilde{G}} - r \right), \quad (7)$$

where $\tilde{G} = G/q_1$, genuine savings expressed in the price of man-made capital. This expression relates saving growth explicitly to the path of utility. If genuine saving is positive and either (i) falling or (ii) rising at a rate lower than the interest rate, then instantaneous utility is rising. If genuine saving is negative and either (i) rising or (ii) falling at a rate lower than the interest rate, then instantaneous utility is falling.

This interpretation can be extended if we assume a constant pure rate of time preference ρ . Then social welfare V and its instantaneous change can be written as⁴

$$V(t) = \int_t^\infty U(x(s), u(s)) e^{-\rho(s-t)} ds \text{ and } \dot{V}(t) = \int_t^\infty \dot{U}(x(s), u(s)) e^{-\rho(s-t)} ds. \quad (8)$$

If $\tilde{G} > 0$ and $\dot{\tilde{G}}/\tilde{G} < r$ everywhere over the unbounded interval $[T, \infty)$, then expression (7) implies that $\dot{U} > 0$ everywhere over the interval; this in turn implies, from expression (8), that $\dot{V} > 0$ at each point in the interval.

In the PV-optimal economy with constant pure rate of time preference we know that $\tilde{G} > 0$ at a point in time implies that $\dot{V} > 0$ at that instant. This is the principal result of Hamilton and Clemens (1999): positive genuine saving is equivalent to increasing social welfare. For the competitive efficient economy the link between saving and social welfare is as follows: if genuine saving is positive and growing at a rate lower than the interest rate over an unbounded interval, then social welfare is everywhere increasing over this interval.

A final remark concerns wealth. If aggregate profits are zero, for example, owing to constant returns to scale, then $p \cdot u + \tau \cdot x + \dot{W} = 0$, where $W = q \cdot x$. Hence,

$$W(t) = \int_t^\infty [p(s) \cdot u(s) + \tau(s) \cdot x(s)] ds.$$

So, total wealth in the economy is the value of all future consumption, in the broad sense, including the possible disutility of pollution stocks. Hamilton and Hartwick (2005) derive a similar result in a less general setting.

4 The latter expression follows by integration by parts.

3. Applications: the Dasgupta-Heal-Solow-Stiglitz economy

The Dasgupta-Heal-Solow-Stiglitz (DHSS) economy with fixed technology and an exhaustible resource that is essential for production presents the canonical problem for sustainability. The solution to finding a sustainable path, rooted in Solow (1974) and derived in Hartwick (1977), lies in the ‘Hartwick rule’: if genuine saving is everywhere zero in a competitive economy, then maximal constant consumption results. We explore two instances of the new saving rule of proposition 2 for this economy, both of which yield unbounded consumption under appropriate conditions on technology and the level of saving.

3.1. A constant net (genuine) saving rate rule

Consider the DHSS economy with a Cobb-Douglas technology $F = K^\alpha R^\beta$, $\alpha + \beta = 1$, where F denotes production, K is man-made capital and R is raw material input from a non-renewable natural resource, whose size is denoted by S . In what follows we work along the lines of (7). In a general equilibrium $pC + q_1\dot{K} + q_2\dot{S} + \dot{q}_1K + \dot{q}_2S$ is maximized, subject to $\dot{K} = K^\alpha R^\beta - C$ and $\dot{S} = -R$. Hence, along an interior equilibrium, we have $p = q_1$, $\dot{q}_2 = 0$, $q_1\alpha F = q_1rK = -\dot{q}_1K$, $q_1\beta F = q_2R$ and the maximum equals zero. For this economy Solow (1974) showed that constant consumption is feasible if and only if $\alpha > \beta$. We explore the following saving rule:

There is a constant γ with $0 < \gamma < \alpha - \beta$, such that $\tilde{G} = \dot{K} + \frac{q_2}{q_1}\dot{S} = \gamma F$.

Along the interior equilibrium this reduces to $\tilde{G} = \dot{K} + F_R\dot{S} = \gamma F$, while efficiency is expressed as the familiar Hotelling rule, $\dot{F}_R = F_R F_K$. This is a constant *net* saving rule, which can be compared to the constant *gross* saving rule discussed in the next subsection. It follows immediately from the saving rule that $C = (\alpha - \gamma)F$, while efficiency yields

$$F_K = \alpha \frac{\dot{K}}{K} - \alpha \frac{\dot{R}}{R}.$$

Note that

$$\frac{\dot{\tilde{G}}}{\tilde{G}} = \frac{\dot{C}}{C} = \frac{\dot{F}}{F}.$$

The saving rule implies that $\dot{K} = (\gamma + \beta)F$, or

$$\frac{\dot{K}}{K} = \frac{\gamma + \beta}{\alpha} F_K,$$

while this expression plus the efficiency condition implies that

$$\frac{\dot{R}}{R} = \frac{\gamma - \alpha}{\alpha} F_K.$$

From the foregoing it follows that

$$\frac{\dot{F}}{F} = \alpha \frac{\dot{K}}{K} + \beta \frac{\dot{R}}{R} = \frac{\gamma}{\alpha} F_K.$$

Since $\alpha > \gamma$, $\tilde{G} > 0$ and therefore

$$\frac{\dot{\tilde{G}}}{\tilde{G}} < F_K,$$

it follows from expression (7) that consumption is everywhere increasing. We can go further and show that consumption is not only increasing but unbounded.

Efficiency implies that

$$\frac{d}{dt} \left(\frac{K}{R} \right) = \frac{1}{\alpha} F_K \frac{K}{R} = \left(\frac{K}{R} \right)^\alpha,$$

which has solution, $K/R = [\beta t + (K_0/R(0))^\beta]^{1/\beta}$. Efficiency also requires that

$$\frac{F}{R} = \frac{F(0)}{R(0)} e^{\int_0^t F_K(s) ds} = \left(\frac{K}{R} \right)^\alpha,$$

implying that $e^{\int_0^t F_K(s) ds} = K_0^{-\alpha} R(0)^\alpha [\beta t + (K_0/R(0))^\beta]^{1/\beta}$.

We therefore conclude that $R = K_0^{\alpha-\gamma} R(0)^{\beta+\gamma} [\beta t + (K_0/R(0))^\beta]^{1/\beta}$. Since it has been assumed that $\gamma < \alpha - \beta$, we have $\int_0^\infty R(t) dt = S_0$ if $R(0) = (\alpha - \beta - \gamma)^{1/\alpha} S_0^{1/\alpha} K_0^{-\beta/\alpha}$. Hence consumption is given by $C = (\alpha - \gamma)F = (\alpha - \gamma)K_0^{\alpha-\gamma} R(0)^{\beta+\gamma} [\beta t + (K_0/R(0))^\beta]^{1/\beta}$, and so is unbounded under the constant genuine saving rate rule. If $\gamma = 0$, then the saving rule reduces to the standard Hartwick rule with constant consumption. Under the assumed condition, $0 < \gamma < \alpha - \beta$, initial consumption on the constant (non-zero) genuine saving path is lower than under the standard Hartwick rule; however, total wealth (the present value of consumption) under the rule is higher, since it is given by

$$W(0) = K_0 + F_R(0)S_0 = \frac{\alpha - \gamma}{\alpha - \beta - \gamma} K_0.$$

Note that wealth is independent of the resource stock size, but does depend upon technology (α). This is a consequence of the sustainability policy rule. The size of the resource endowment affects $R(0)$ and therefore $F_R(0)$ in such a way as

to render total wealth invariant to resource stock size. In particular, this would mean that two autarkic DHSS economies with the same initial capital stock and technology but differing resource endowments would have the same wealth *measured in local prices* under this policy rule.

Income growth is a typical concern for policymakers. Since Hicksian income $C + \tilde{G}$ equals αF under the constant genuine saving rate rule, it follows that

$$\frac{\frac{d}{dt}(C + \tilde{G})}{C + \tilde{G}} = \gamma \cdot \frac{F_K}{\alpha}.$$

If the long-run real rate of return on capital is 3.5% and $\alpha = 0.7$, both plausible values, then a 1% rise in the genuine saving rate will boost income growth by 0.05%.

A few final remarks are in order. First, the feasibility of increasing consumption in this economy was shown by Dasgupta and Heal (1979, 307) for an optimal path with zero rate of time preference and a utilitarian welfare function with an instantaneous utility function having constant elasticity of marginal utility (subject to some restrictions on the parameters). Second, Mitra (1983) showed that total consumption will be unbounded in the DHSS economy if it follows a particular quasi-arithmetic path.⁵ We have shown that such a path results naturally from the constant saving rate rule. Third, the formula describing the time path of consumption suggests that with a particular function describing population growth, with population increasing to infinity but at a decreasing rate, indefinite positive per capita consumption can be sustained. This is further explored in Asheim et al. (2005).

3.2. A constant gross saving rate rule

Asheim and Buchholz (2004) derive several of the properties of a constant *gross* saving rate rule, given by $\dot{K} = \omega F$, in a DHSS economy. It is straightforward to show, substituting $\gamma + \beta$ from the preceding model for ω in the Asheim-Buchholz model,⁶ that

$$\frac{\dot{\tilde{G}}}{\tilde{G}} = \frac{\omega - \beta}{\alpha} F_K,$$

and therefore that consumption is everywhere increasing in this model under the feasibility assumption ($\omega > \beta$) made by Asheim and Buchholz. Similarly, consumption in the constant gross saving rate model is given by

$$C = (1 - \omega)F = (1 - \omega)K_0^{1-\omega} R(0)^\omega [\beta t + (K_0/R(0))^\beta]^{\frac{\omega-\beta}{\beta}},$$

5 Note that Mitra (1983) actually proved the feasibility of constant per capita consumption in an exhaustible resource economy where population grows at a quasi-arithmetic rate.

6 Recall that in the constant net saving rate model $\tilde{G} = \gamma F \Rightarrow \dot{K} = \gamma F + F_R R = (\gamma + \beta)F$

which is therefore unbounded under the assumed feasibility condition, a result not derived in Asheim and Buchholz (2004).

4. Conclusions

We have extended Dixit, Hammond, and Hoel (1980) to a more general model with potential externalities, first for an optimal economy and second for a decentralized economy that is competitive and efficient. We establish a corollary to the main result of Dixit, Hammond, and Hoel (1980), deriving the relationship between the change in instantaneous utility and the change in genuine saving. For the optimal economy the result is descriptive, in the sense that observing the rate of change of genuine saving can tell us whether instantaneous utility is increasing or decreasing.

For the competitive efficient economy it is possible that a policy rule, specifying positive genuine saving growing at a rate less than the interest rate, could result in utility that grows at each point in time (i.e. sustainability) along the development path. We show that such a policy rule is feasible in the case of the DHSS economy with a constant net or gross saving rate.

The constant genuine saving rate rule results in unbounded consumption, a result that we relate to Dasgupta and Heal (1979) and Mitra (1983). Mitra showed that total consumption will be unbounded in the DHSS economy if it follows a particular quasi-arithmetic path. We show that such a path results naturally from the constant saving rate rule. In this sense our result is not unlike that of Hartwick (1977), who showed that the Solow (1974) model of constant consumption resulted from a simple saving rule. The Hartwick rule is a special case of our constant genuine saving rate rule.

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